BAO detection methods

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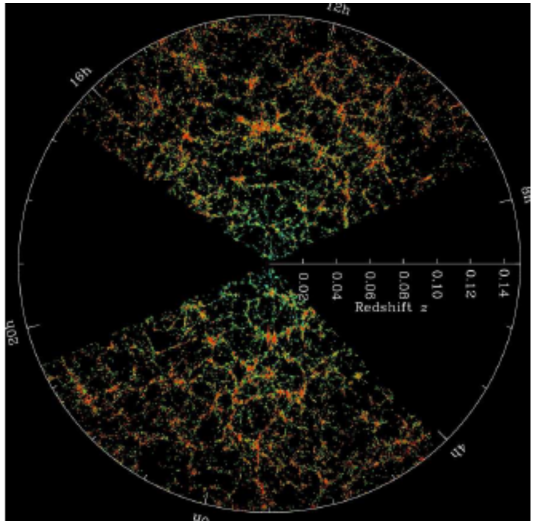
Plan

- Introduction
- BAO detection using Wavelets
- BAO detection with matched filter
- BAO detection with χ^2 method

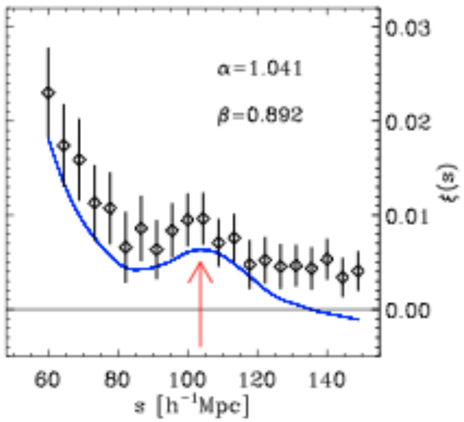
Introduction

Study of the galaxy distribution

- Galaxy surveys become very large
  - Testing cosmological model (ΛCDM)
  - Constraints on cosmic parameters (precision cosmology)



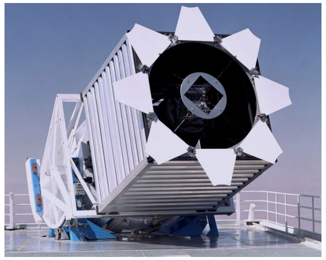
- Method:
  - Comparison with N-body simulations
  - Comparison with theoretical predictions



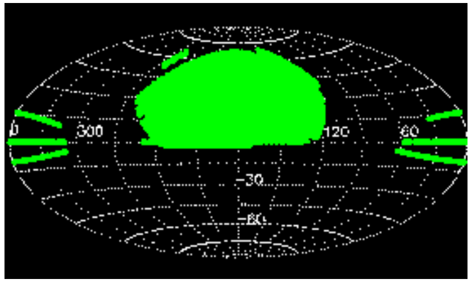
Fitting model correlation function to the data

SDSS galaxy survey DR7

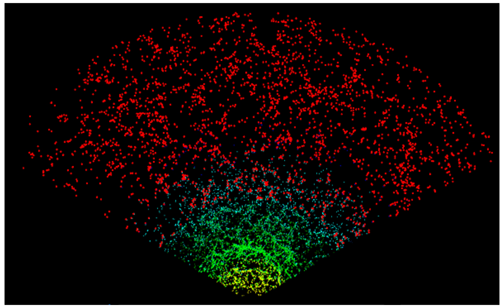
- 8 year program with 2.5m telescope at Apache Point (New Mexico)



  - Mapped 7500 square degree of the sky
  - Spectrum for 930 000 galaxies (largest galaxy survey up to date)



  - 1 magnitude-limited samples of galaxies ***(Main)*** up to D ≈ 600 h^{-1}Mpc
  - 1 approximately volume-limited of luminous red galaxies ***(LRG)*** up to D ≈ 1150 h^{-1}Mpc

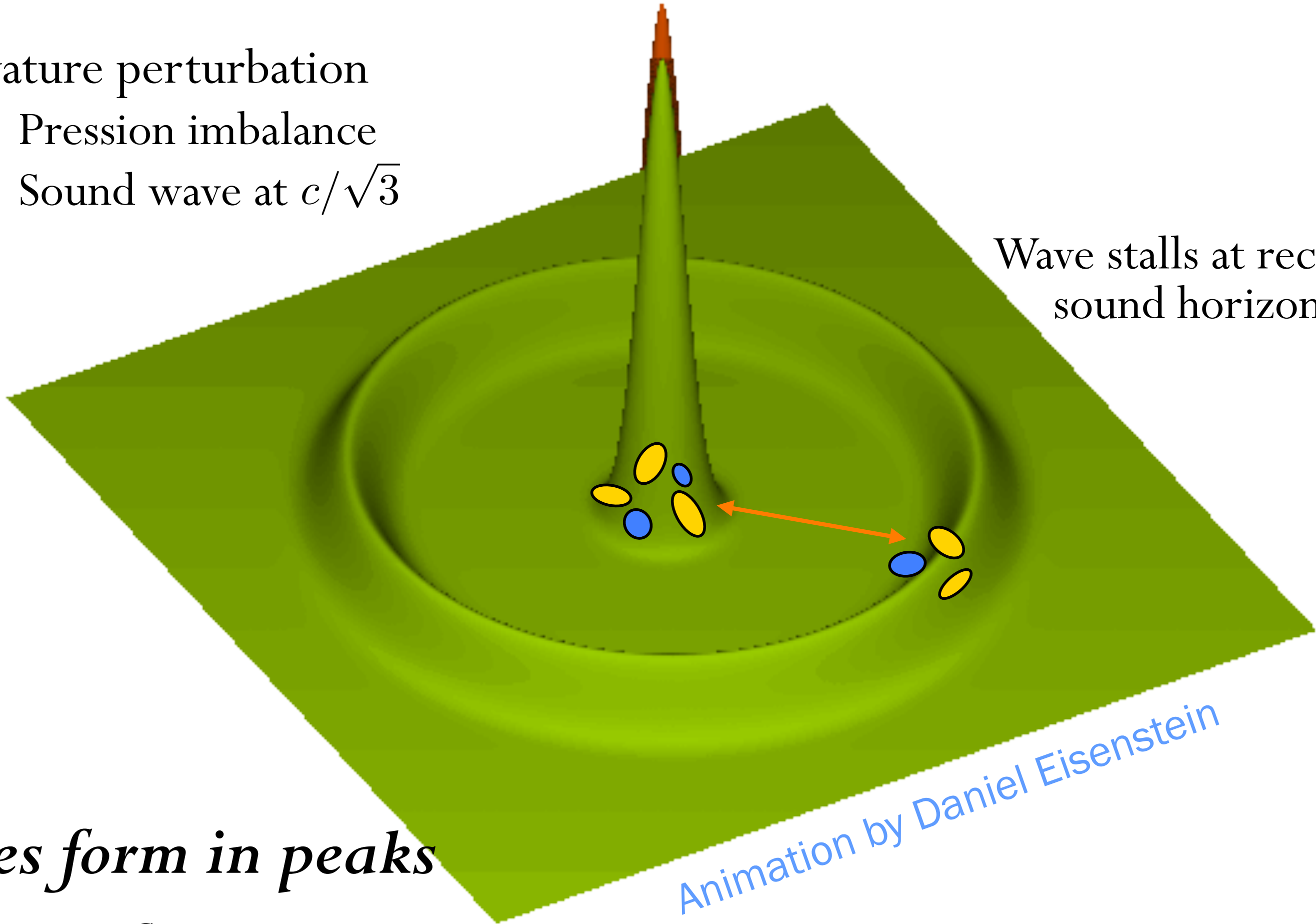


Baryon Acoustic Oscillations (I)
Photon Pressure supported waves

Curvature perturbation

- Pression imbalance
- Sound wave at $c/\sqrt{3}$

Wave stalls at recombination with sound horizon r_s=150 Mpc



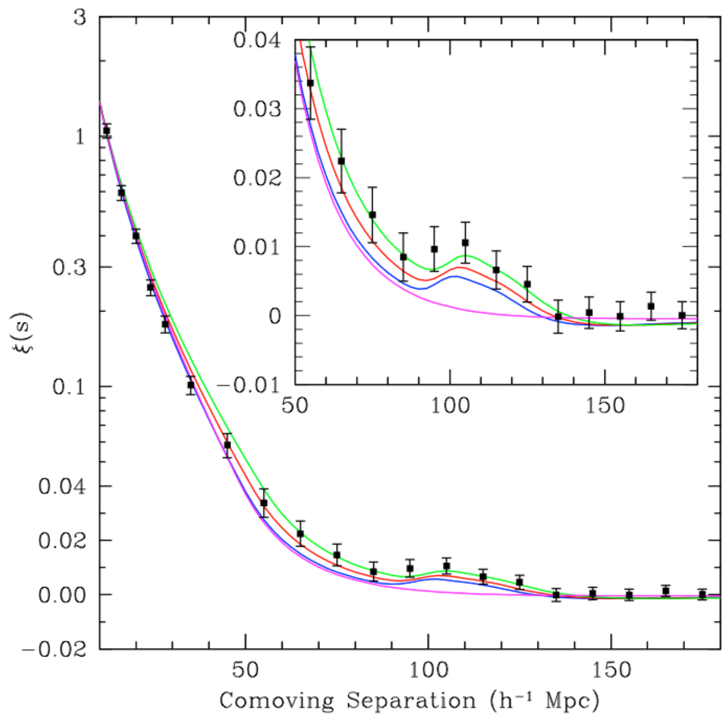
Galaxies form in peaks

- ***Excess of correlation***

BAO in correlation function $\xi(r)$

- Dependence of ξ on cosmological parameters:
  - Amplitude of BAO peak:
    -mass baryon Ω_b *and matter* Ω_m
  - Apparent location of BAO peak:
    -distance conversion in redshift catalogues
  - Amplitude of correlation:
    -bias of galaxy correlation ξ_g *compared to dark matter* ξ_{dm}

- BAOs enable to constrain cosmological parameters ***but first*** they must be detected



Different ξ curves with $\Omega_m h^2 = 0.12, 0.13, 0.14$ (green, red, blue) and non physical no-BAO model (pink) (Eisenstein et al. 2005)

BAO detection methodology

- We will write:

  H_0: no-BAO hypothesis

  H_1: BAO hypothesis

- 2 steps in the detection:
  - Rejection of H_0 → ***Subject of this talk***

    -Significant deviation of a statistic compared to its expected value under H_0

    -Independent of H_1

    -H_1 *only helps to find statistic with strong difference under* H_0 *and* H_1

  - Test of compatibility with H_1

I) BAO detection with wavelets

H_0: No significant bump in the correlation function
H_1: BAOs create a significant localized bump in the correlation function

Method with very little assumptions

BAOlet method (I)

Arnalte-Mur et al. 2010 (arXiv:1101.1911)

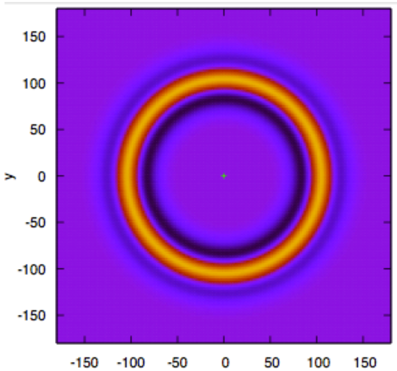
- SDSS **Main** density field δ convolved with spherical wavelet (BAOlet)

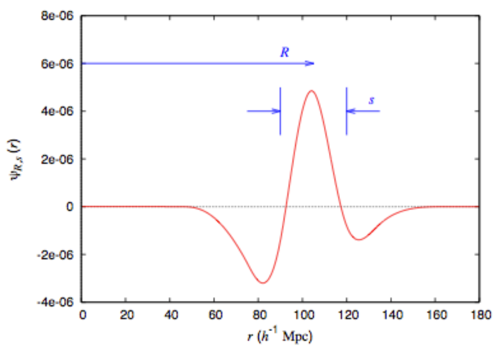
$$W_{R,s}(\vec{x}) = (\Psi_{R,s} * \delta)(\vec{x})$$

- We look at the mean at SDSS **LRG** positions

$$B(R,s) = \left\langle W_{R,s}(\vec{x}^{(i)}_{LRG}) \right\rangle_i$$

- BAOlet controlled by 2 parameters:
  - R : distance from the center to the acoustic shell
  - s : width of the shell





BAOlet method (II)

- Equivalent with 1D wavelet transform on ***cross-correlation LRG-Main***

- Calculate noise $\sigma(R,s)$ in $B(R,s)$ using bootstrap or galaxy mock catalogues

- Z-score on the data and on the simulations

$$Z^D(R,s) = \frac{B^D(R,s)}{\sigma(R,s)} \qquad Z^s(R,s) = \frac{B^s(R,s)}{\sigma(R,s)}$$

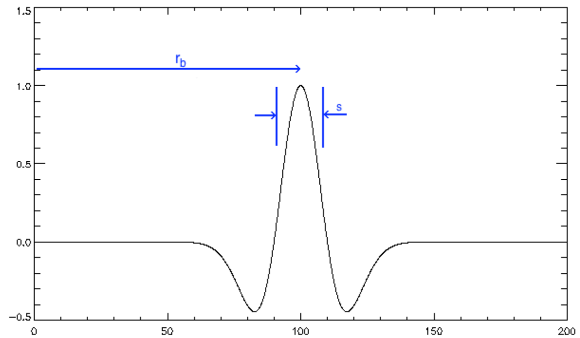
- Calculate $Z^D(R_{max},s_{max})$ at the maximum (R_{max},s_{max}) of $B(R,s)$ and obtain p-value with simulations

$$p = P\left[Z^s(R^s_{max}, s^s_{max}) \geq Z^D(R_{max}, s_{max})\right]$$

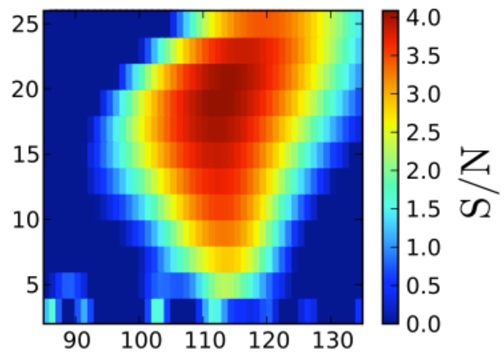
Other wavelet method

Tian et al. 2010

- Mexican hat wavelet:
  - r_b : distance from 0 to the peak
  - s : width of the function



- 1D transform on the correlation function of SDSS Main
- Rejection of $\boldsymbol{H_0}$ with 3σ significance *(p-value corresponding to 3σ deviation for a gaussian)*



II) BAO detection with matched filter

H_0: Correlation function ξ_{noBAO}

H_1: Correlation function ξ_{BAO}

Optimal filtering method for fixed cosmological parameters

Construction of the matched filter (I)

- Look for a BAO signal by filtering the correlation function of the data

$$S_w = \left\langle \hat{\xi}, w \right\rangle = \sum_{i=1}^{r} \hat{\xi}(r_i) w_i$$

- Expected value under the different hypotheses

$$H_0 : \mathbb{E}(S_w) = \langle \xi_{noBAO}, w \rangle$$

$$H_1 : \mathbb{E}(S_w) = \langle \xi_{BAO}, w \rangle$$

- Noise $\sigma[S_w]$ given with covariance matrix **C** (estimated on simulations) of $\hat{\xi}$:

$$\sigma[S_w] = \langle w, Cw \rangle$$

Construction of the matched filter (II)

- We define the signal-to-noise

$$\mathrm{S}NR_w = \frac{\left\langle \hat{\xi} - \xi_{noBAO}, w \right\rangle}{\langle w, Cw \rangle}$$

- Under $\boldsymbol{H_0}$:

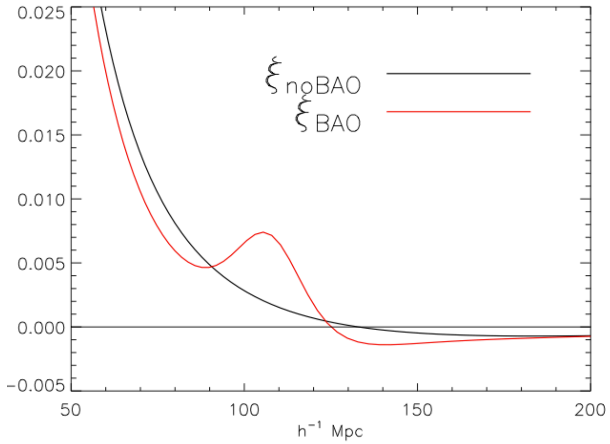
$$\mathbb{E}[\mathrm{S}NR_w] = 0$$

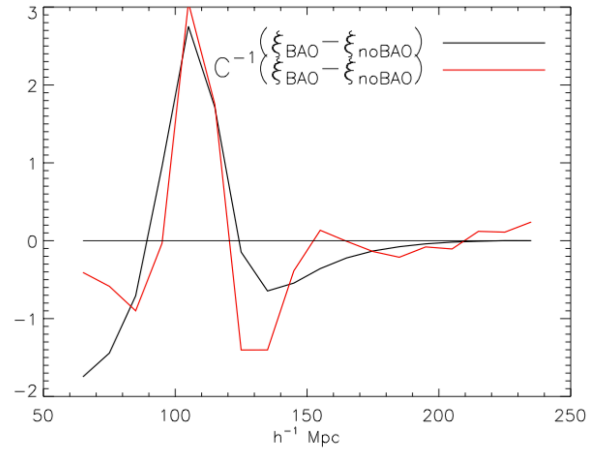
$$\sigma[\mathrm{S}NR_w] = 1$$

- Optimal filter defined to maximize the expected signal-to-noise under $\boldsymbol{H_1}$:

$$w = C^{-1}\left(\xi_{BAO} - \xi_{noBAO}\right)$$

SDSS LRG simulations





- $w=\xi_{BAO} - \xi_{noBAO}$ → $\mathbb{E}[SNR_w] = 2.05$
- Best BAOlet *(R=107 h^{-1}Mpc, s=31h^{-1}Mpc)* → $\mathbb{E}[SNR_w] = 2.42$
- Best Mexican hat *(R=107 h^{-1}Mpc, s=13h^{-1}Mpc)* → $\mathbb{E}[SNR_w] = 2.41$
- Matched filter $w=C^{-1}(\xi_{BAO} - \xi_{noBAO})$ → $\mathbb{E}[SNR_w] = 2.46$

III) BAO detection with χ^2 method

H_0: Correlation function in the class $\xi_{noBAO}(\theta)$

H_1: Correlation function in the class $\xi_{BAO}(\theta)$

Rigorous method which allows variations of cosmological parameters

χ^2 statistic

- For a model correlation function ξ_m

$$\chi^2 = \left\langle \left(\hat{\xi} - \xi_m\right), C^{-1}\left(\hat{\xi} - \xi_m\right)\right\rangle$$

- If the ***model is true*** and ***measurement is gaussian***, χ^2 follows chi-square distribution with *n* degrees of freedom (*n* number of bins in the correlation vector)

$$\chi^2 \sim \chi^2_n$$

- For a class $\xi_m(\theta)$ with ***k*-dimensional** parameter $\theta = (\theta_1, \ldots, \theta_k)$

$$\chi^2(\theta) = \left\langle \left(\hat{\xi} - \xi_m(\theta)\right), C^{-1}\left(\hat{\xi} - \xi_m(\theta)\right)\right\rangle$$

- If the true model is inside the class (with assumption that models are linear wrt θ)

$$\min_{\theta} \chi^2(\theta) \sim \chi^2_{n-k}$$

BAO detection with χ^2 (I)

- Under ***H$_0$***:

$$\min_{\theta} \chi^2_{noBAO}(\theta) \sim \chi^2_{n-k}$$

- Create an artificial extended model

$$\xi_{ext}(\theta, \alpha) = \alpha \xi_{BAO}(\theta) + (1 - \alpha)\xi_{noBAO}(\theta)$$

- Under ***H$_0$***, difference of best fits in restricted and extended model (again with assumption that models are linear wrt θ)

$$\Delta\chi^2_{ext} = \min_{\theta} \chi^2_{noBAO}(\theta) - \min_{\theta,\alpha} \chi^2_{ext}(\theta, \alpha)$$

$$\Delta\chi^2_{ext} \sim \chi^2_1$$

BAO detection with χ^2 (II)

- Let us note $\Delta\chi^2 = \min_\theta \chi^2_{noBAO}(\theta) - \min_\theta \chi^2_{BAO}(\theta)$

- Since the BAO model is in the extended model

$$\Delta\chi^2 \leq \Delta\chi^2_{ext}$$

- $\Delta\chi^2$ is less than a $\chi_1{}^2$ variable, so under $\boldsymbol{H_0}$

$$P(\Delta\chi^2 \geq x) \leq P(X^2 \geq x) \quad \textit{with } X \sim \mathcal{N}(0,1)$$

- Conservative significance level given by $\sqrt{\Delta\chi^2}$

$\Delta\chi^2$ and matched filter

- For a fixed cosmology, under $\boldsymbol{H_1}$:
  - Conservativity χ^2 method: $\mathbb{E}\left[\sqrt{\Delta\chi^2}\right] \approx \mathbb{E}\left[SNR_w\right] - \frac{1}{2\,\mathbb{E}\left[SNR_w\right]}$
  - SDSS LRG with $SNR_w = 2.5$: $\mathbb{E}\left[\sqrt{\Delta\chi^2}\right] \approx \mathbb{E}\left[SNR_w\right] - 0.2$

    on simulations: 2.23 2.46
  - Significance with $\Delta\chi^2_{ext}$ $\mathbb{E}\left[\sqrt{\Delta\chi^2_{ext}}\right] \approx \mathbb{E}\left[SNR_w\right]$
- For class of cosmologies, under $\boldsymbol{H_1}$:
  - Difficult to extent matched filter method
  - But we still have: $\mathbb{E}\left[\sqrt{\Delta\chi^2}\right] \approx \mathbb{E}\left[\sqrt{\Delta\chi^2_{ext}}\right] - \frac{1}{2\,\mathbb{E}\left[\sqrt{\Delta\chi^2_{ext}}\right]}$

Conclusion

Conclusion

- 3 classes of method for BAO detection by rejecting $\boldsymbol{H_0}$:
  - Wavelet methods assume negligible response to peak finder
  - Matched filter optimal for fixed cosmological parameters
  - χ^2 method rigorous for variations in cosmological parameters

- χ^2 conservative → underestimates significance (better result with matched filter for fixed cosmological parameters)

- χ^2 method can be easily modified (extended artifical model) to obtain better significance even with varying cosmological parameters