A new method for detecting Baryon Acoustic Oscillations

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Physics of BAOs

- Competition between gravitation and photon pressure in the primordial plasma → sound waves excited at speed $c/\sqrt{3}$
- Wave stops at recombination at sound horizon scale $\mathbf{r_s} \approx \mathbf{150Mpc}$ (well constrained by CMB)
- Galaxies form in matter density peaks → excess of correlation at that scale

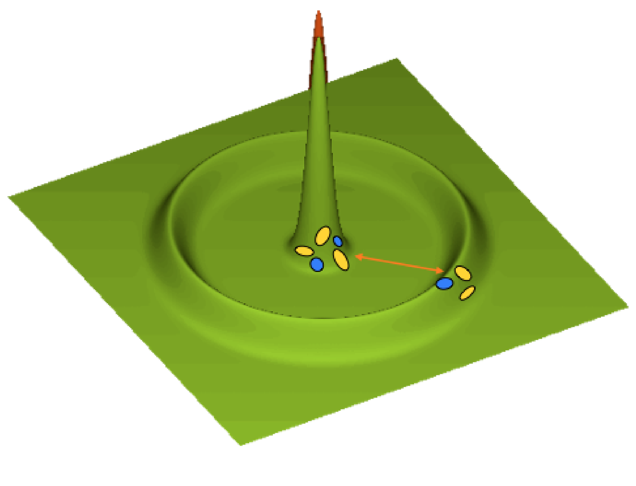
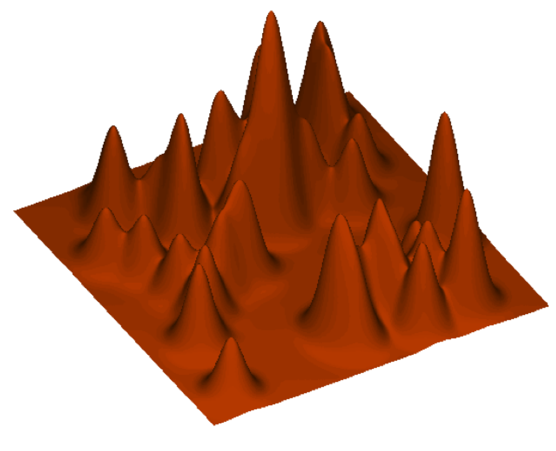


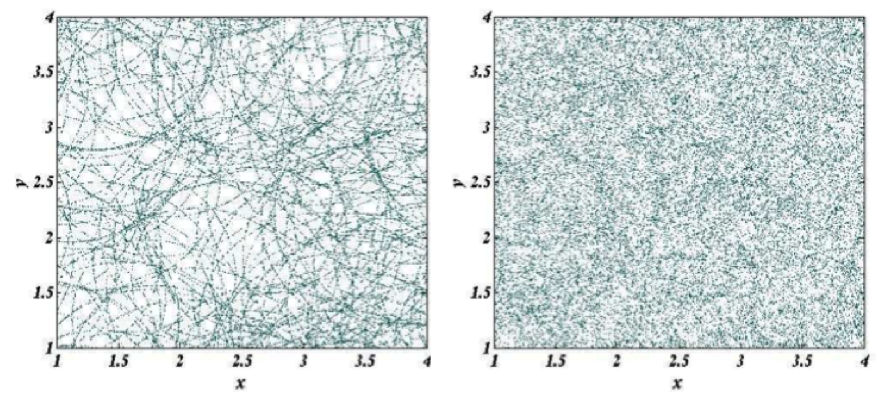
Figure by Daniel Eisenstein

BAOs as standard ruler (I)

- In reality, waves originate from everywhere and superpose
- Only a 1% statistical effect
- Can only be seen statistically → requires a large volume



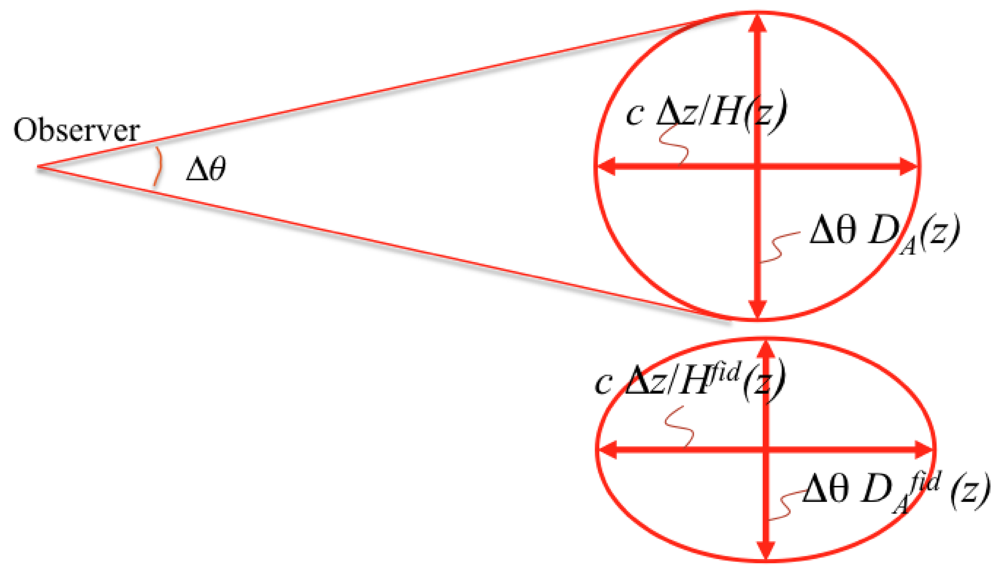
Superpositon of waves. Figure by Daniel Eisenstein.



Left: Very dense rings of galaxies superposed. Right: Less dense rings superposed. The scale of the rings can only be recovered statistically. Figure from Bassett & Hlozek 2009.

BAOs as standard ruler (II)

- Galaxy surveys are redshift surveys → one must assume a **fiducial cosmology** to convert redshift into distances and obtain 3D volume



Top: Real object for redshift separation Δz and angle on the sky $\Delta\theta$. Bottom: Distorted object as observed in the fiducial cosmology.

- BAOs are a **standard ruler**, i.e. have a know real size → show how incorrect the fiducial cosmology is

Standard ruler distorted by wrong fiducial cosmology.

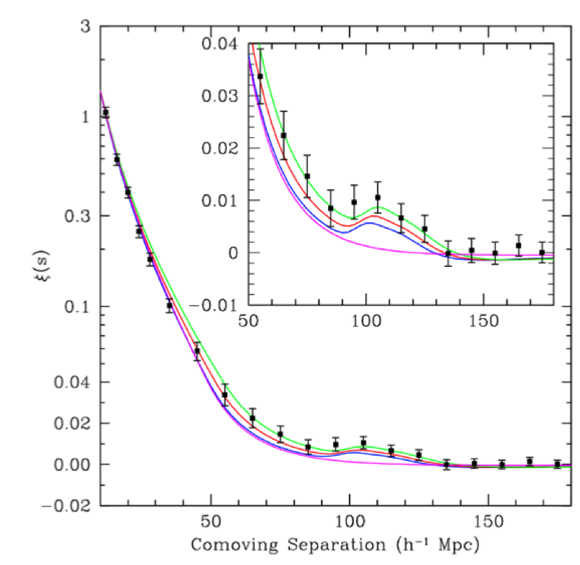
BAOs in the correlation function

3 parameters in the correlation function $\theta = (\Omega_m h^2, \alpha, b)$

- $\Omega_m h^2$ controls global shape of the correlation and amplitude of BAO peak
- α gives a dilation of the correlation function due to incorrect fiducial cosmology
- b^2 is a constant amplitude factor

→ **Detection of BAOs at expected scale confirms cosmological paradigm**

→ **BAOs constrain parameter α with the standard ruler property**



Correlation functions with $\Omega_m h^2 = 0.12, 0.13, 0.14$ (green, red, blue) and non physical no-BAO model (pink). Figure from Eisenstein et al. 2005.

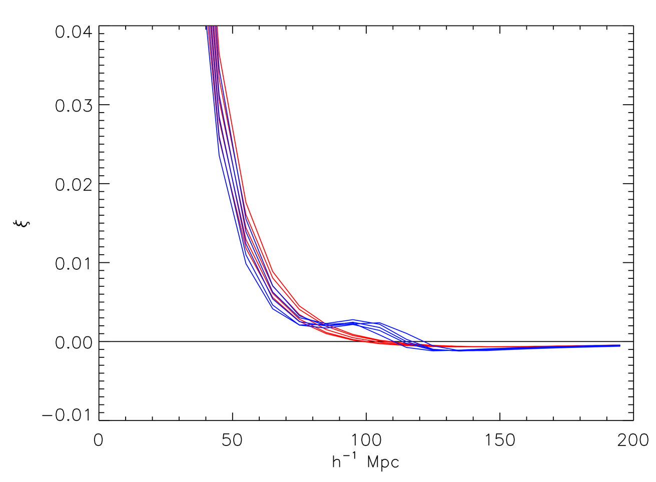
Question

HOW DO WE DETECT BAOs ?

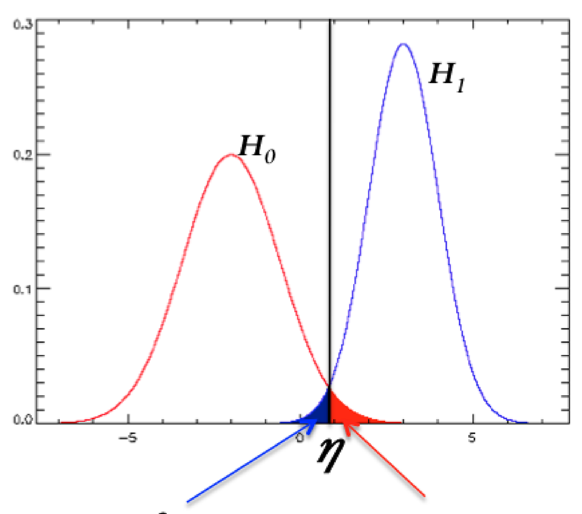
BAO detection = Hypothesis testing

$$\mathcal{H}_0 : \exists\theta \in \Theta \text{ such that } \hat{\xi} \sim \mathcal{N}(\xi_{noBAO,\theta}, C_{noBAO,\theta})$$
$$\mathcal{H}_1 : \exists\theta \in \Theta \text{ such that } \hat{\xi} \sim \mathcal{N}(\xi_{BAO,\theta}, C_{BAO,\theta})$$

- Statistical test of size α with *test statistic* $t(\hat{\xi})$
  - if $t(\hat{\xi}) > \eta$ then accept $\mathcal{H}_1$
  - if $t(\hat{\xi}) \leq \eta$ then accept $\mathcal{H}_0$
- More common in cosmology
  → **Significance = proba under $\mathcal{H}_0$ that $t(\hat{\xi}) >$ observed value**



Different $\xi_{noBAO,\theta}$ in $\mathcal{H}_0$ (red) and $\xi_{BAO,\theta}$ in $\mathcal{H}_1$ (blue).



probability β under H_1 that H_1 is rejected — *Probability α under H_0 that H_0 is rejected*

Classical method for BAO detection

- Assumes constant covariance matrix $C = C_{BAO,\theta} = C_{noBAO,\theta}$
- *test statistic* = generalized likelihood ratio $\Delta\chi^2(\hat{\xi})$

$$\Delta\chi^2 = \min_\theta \chi^2_{noBAO,\theta} - \min_\theta \chi^2_{BAO,\theta} = -2\log\left[\frac{\max_\theta \mathcal{L}_{noBAO,\theta}}{\max_\theta \mathcal{L}_{BAO,\theta}}\right]$$

- Large values of $\Delta\chi^2$ favor $\mathcal{H}_1$
  → **Significance = proba under $\mathcal{H}_0$ that $\Delta\chi^2 >$ observed value**
- Given some assumptions: $\Delta\chi^2 \leq X^2$ with $X \sim \mathcal{N}(0,1)$ under $\mathcal{H}_0$
- Significance can be estimated as $P(X^2 > \Delta\chi^2)$ i.e. as $\sqrt{\Delta\chi^2}.\sigma$

Problems

- Regularity assumptions are usually not verified → $\sqrt{\Delta\chi^2}.\sigma$ **overestimates the significance**
- The method can work only for constant covariance matrix C and **not for model-dependent $C_{noBAO,\theta}$, $C_{BAO,\theta}$**

New method for BAO detection: Δl method

- New procedure to estimate significance (works in all cases)
  - We generate realizations of every model θ in $\mathcal{H}_0$
    $$\hat{\xi} \sim \mathcal{N}(\xi_{noBAO,\theta}, C_{noBAO,\theta})$$
  - We compute the significance for every $\mathcal{H}_0$ model and every x
    $$P(\Delta\chi^2 \geq x \,|\, \mathcal{H}_0, \theta)$$
  - BAO detection significance given by the 'worst case' $\mathcal{H}_0$ model:
    $$p(x) = \max_\theta P(\Delta\chi^2 \geq x \,|\, \mathcal{H}_0, \theta)$$
- Instead of $\Delta\chi^2$ we use Δl, which is still a generalized likelihood ratio for model-dependent covariance matrix

$$\Delta l = \min_\theta l_{noBAO,\theta} - \min_\theta l_{BAO,\theta}$$
$$l_{BAO,\theta} = \chi^2_{BAO,\theta} + \log|C_{BAO,\theta}|$$
$$l_{noBAO,\theta} = \chi^2_{noBAO,\theta} + \log|C_{noBAO,\theta}|$$

Results

- We use lognormal simulations of the SDSS DR7 LRG sample
  → deduce constant covariance matrix C and model-dependent covariance matrix C_θ
- We test the different BAO detection methods

	Classical $\sqrt{\Delta\chi^2}.\sigma$ (wrong)	$\Delta\chi^2$ with correct significance	Δl method
Constant cov matrix C	2.21σ	2.0σ	2.0σ
Model-dependent cov matrix C_θ	2.32σ	1.59σ	1.96σ

Table: Mean significance obtained on $\mathcal{H}_1$ realizations in the two different cases of constant C and model-dependent C_θ

$\sqrt{\Delta\chi^2}.\sigma$ slightly overestimates significance for constant C

$\sqrt{\Delta\chi^2}.\sigma$ grossly overestimates significance for model-dependent C_θ

Δl *statistic* largely outperforms $\Delta\chi^2$ *statistic* for model-dependent C_θ