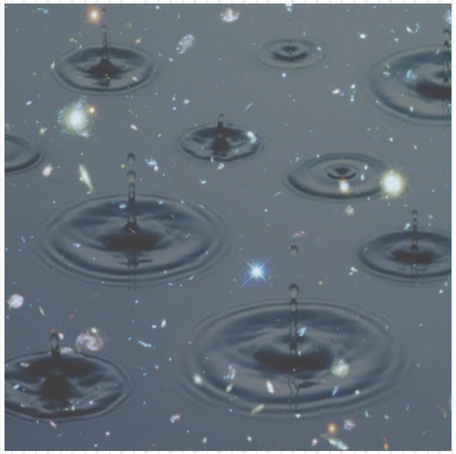
BAO study with model-dependent covariance matrix C_θ



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Plan

I) Introduction on BAOs

II) SDSS survey and simulations

III) BAO detection

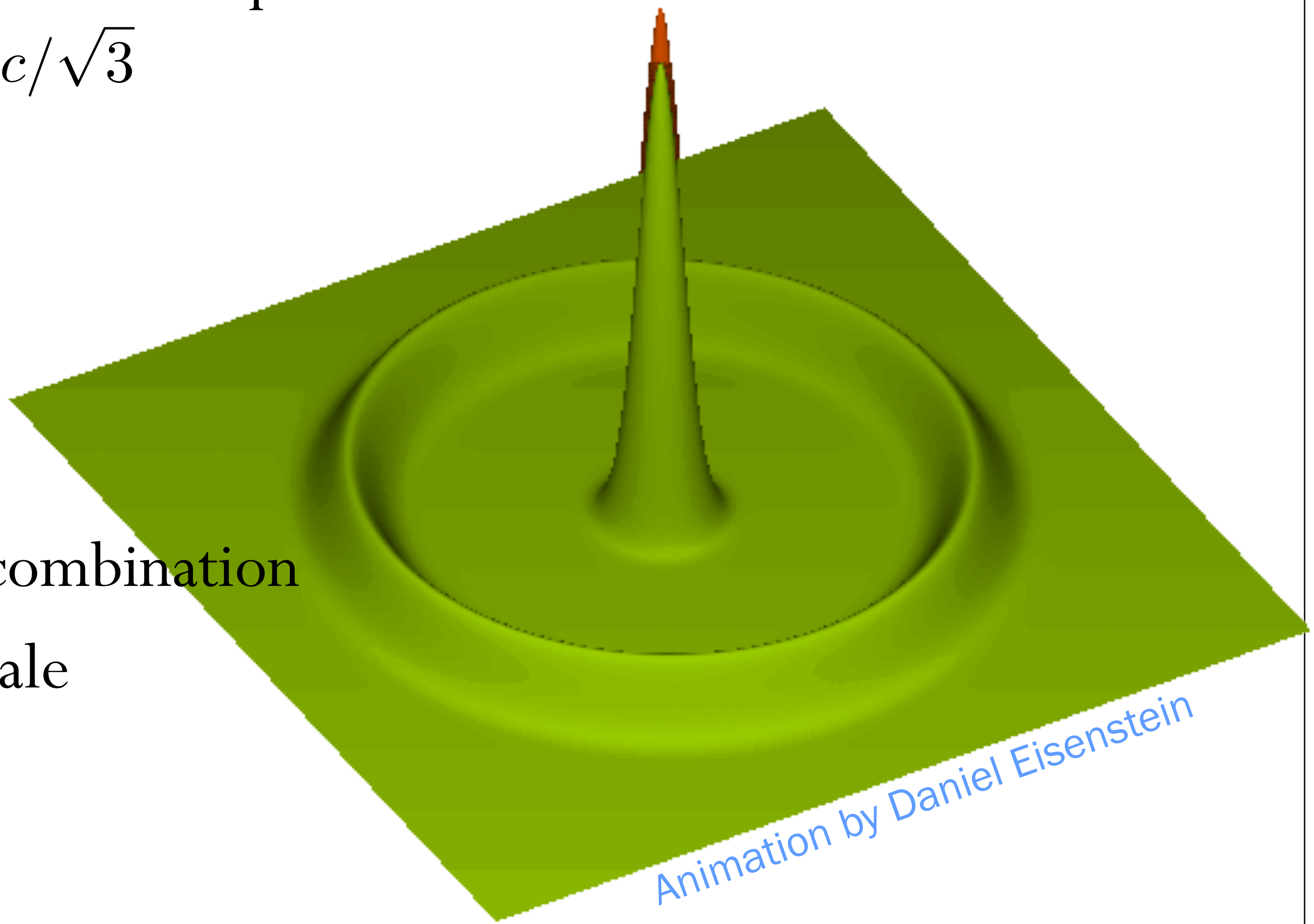
IV) BAOs for cosmological parameter constraints

Conclusion

I) Introduction on Baryon Acoustic Oscillations

Baryon Acoustic Oscillations (I)

- Sound wave excited in the primordial plasma at speed $c/\sqrt{3}$



- Wave stops at recombination

at sound horizon scale

$r_s \approx 150$ Mpc

Baryon Acoustic Oscillations (II)

- Waves originate from everywhere and superpose
- Only **1%** statistical effect

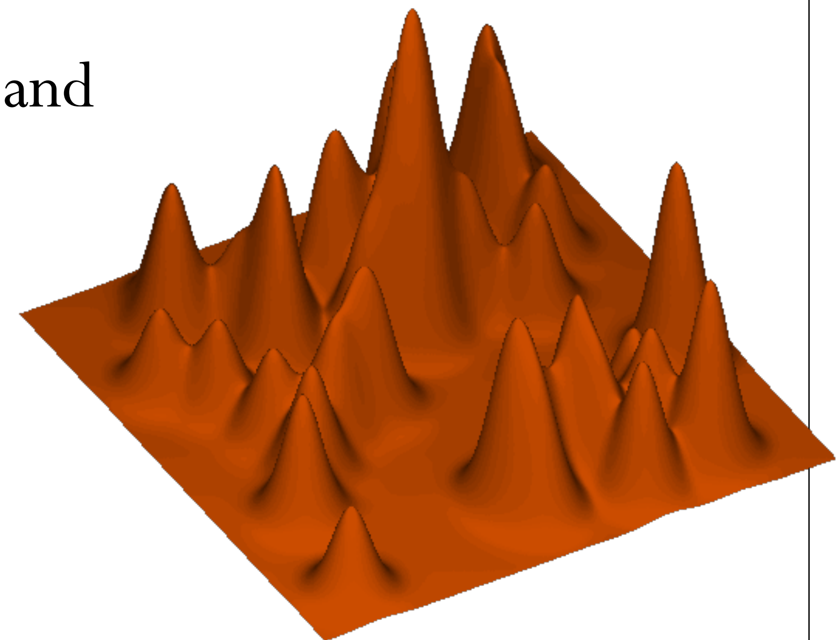


Figure from Daniel Eisenstein

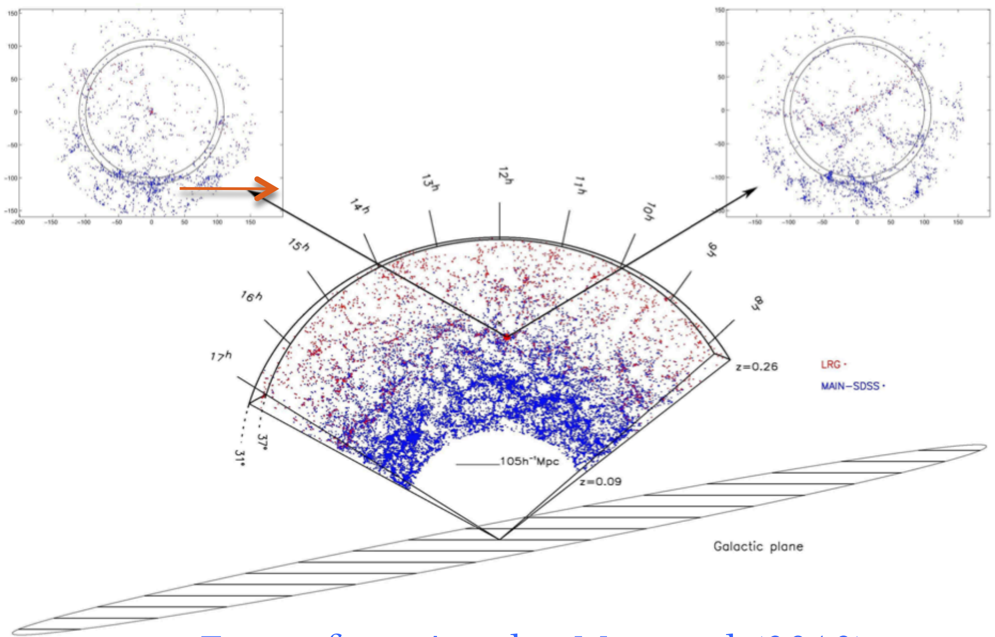


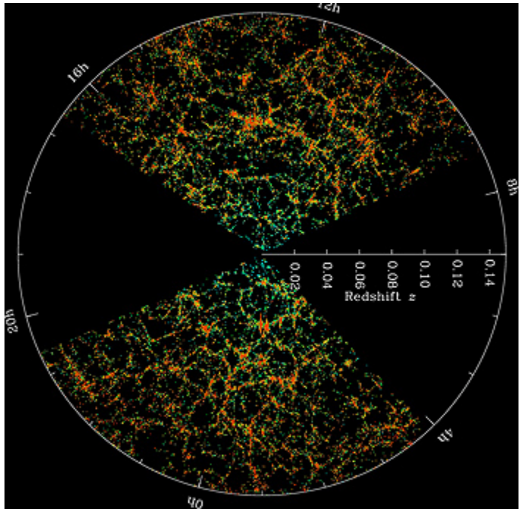
Figure from Arnalte-Mur et al (2012)

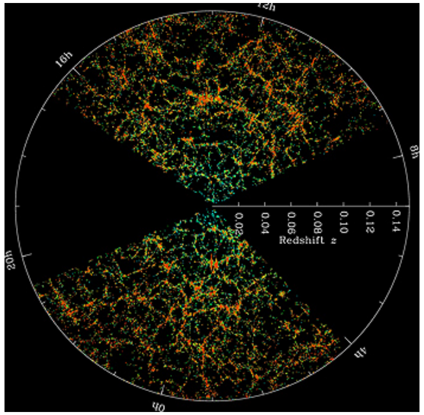
Gravitational evolution to large-scale structures

BAOs = standard ruler

- Galaxy surveys are redshift surveys → one **assumes a fiducial cosmology** to obtain 3D volume: $D_C = D_H \int_0^z \frac{dz'}{E(z')}$

Correct fiducial cosmology

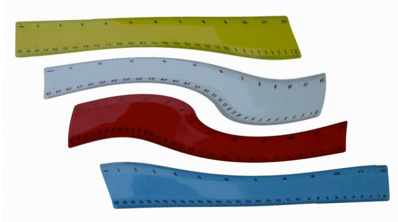




Wrong fiducial cosmology
≈ dilation factor α

- BAOs give a **standard ruler** (known real size)

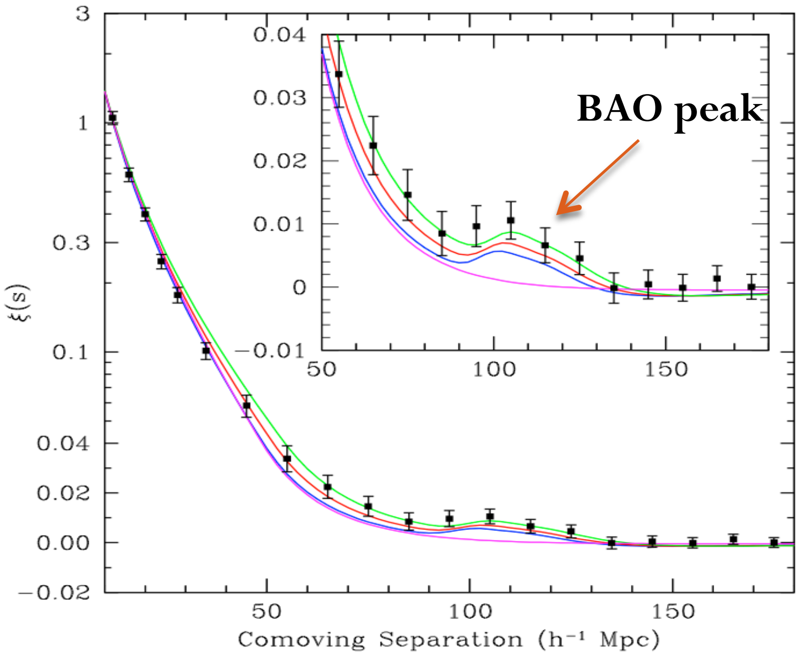
→ they show how incorrect the fiducial cosmology is



BAOs in correlation function ξ(r)

- We restrict to 3 parameters:
  - Geometrical distortion effect
    - ≈ dilation factor $\boldsymbol{\alpha}$ probing **dark energy**
  - Matter density $\boldsymbol{\Omega_m h^2}$
  - Effect of σ_8 and galaxy bias
    - ≈ amplitude factor $\boldsymbol{b^2}$

$$\boxed{\xi_\theta(r) = b^2 \xi_{\Omega_m h^2}(\alpha r)}$$



ξ curves with $\Omega_m h^2 = 0.12, 0.13, 0.14$)
(Eisenstein et al. 2005)

→ **Detection of BAOs at expected scale confirms ΛCDM model**

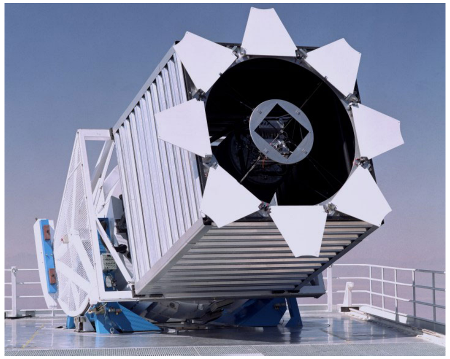
(theoratical prediction by Peebles in 1970)

→ **BAOs constrain $\boldsymbol{\alpha}$ with standard ruler property**

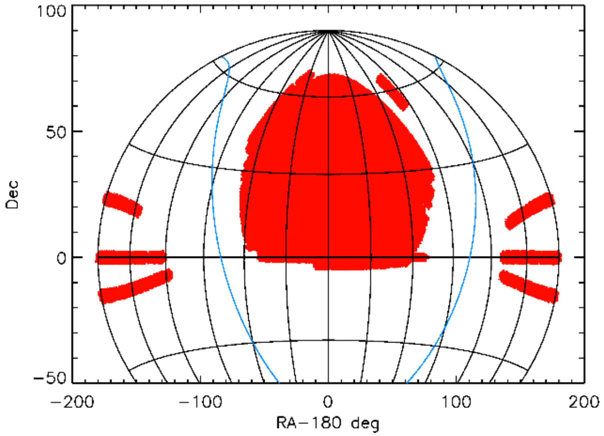
II) SDSS survey and simulations

SDSS LRG survey

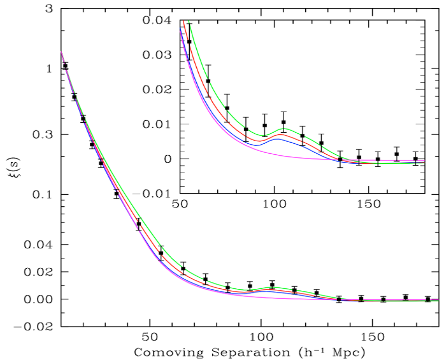
- 8 year program with 2.5m telescope at Apache point (New Mexico)
- SDSS DR7 in 2008, last release of SDSS II
- BOSS DR9 only public in July 2012



- Sample used: (Kazin et al. 2010)
  - Spectroscopic LRG sample with 105k galaxies
  - Redshift range: 0.16<z<0.47



SDSS DR7 simulations



$$\exists \theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}(\xi_\theta, C_\theta)$$

CAN WE REALLY APPROXIMATE AS A CONSTANT?

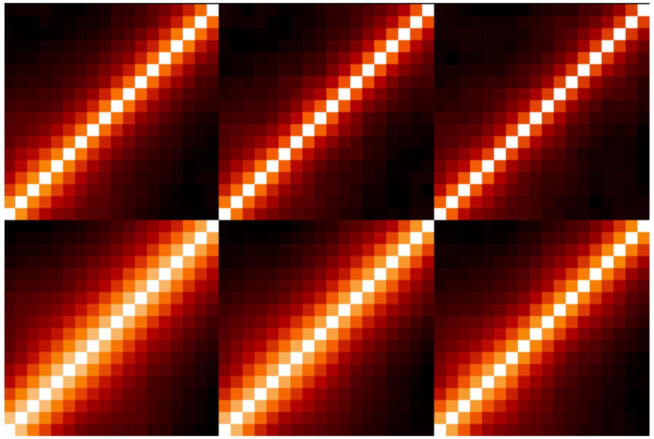
- We need ≈ 2000 lognormal simulations for each θ **(very time consuming)**

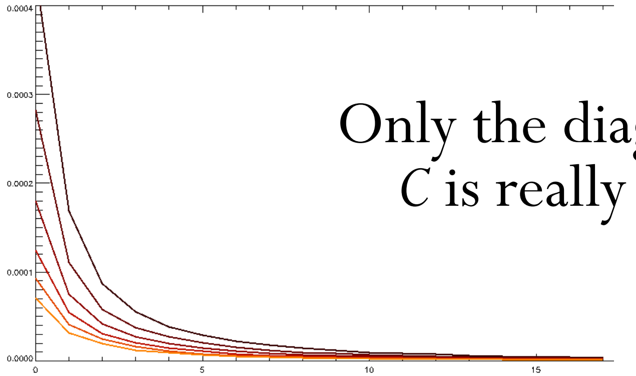
- We have developed new fast technique to obtain realistic covariance matrices depending on $\boldsymbol{\theta=(\alpha, \Omega_m h^2, b)}$

- **We use our simulations only to get model-dependent C_θ**

Structure of C_θ

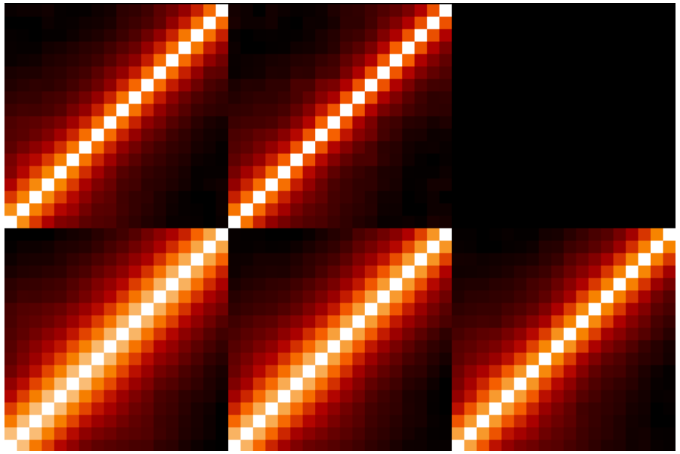
Effect of α

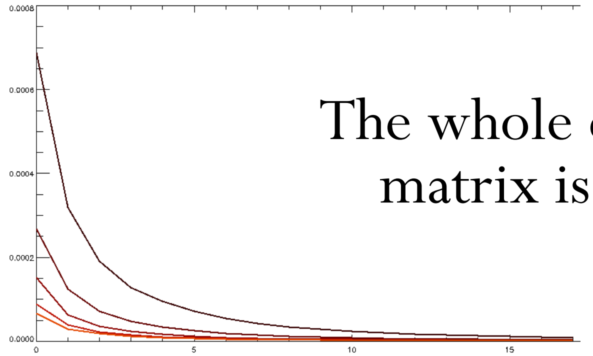




Only the diagonal of C is really affected

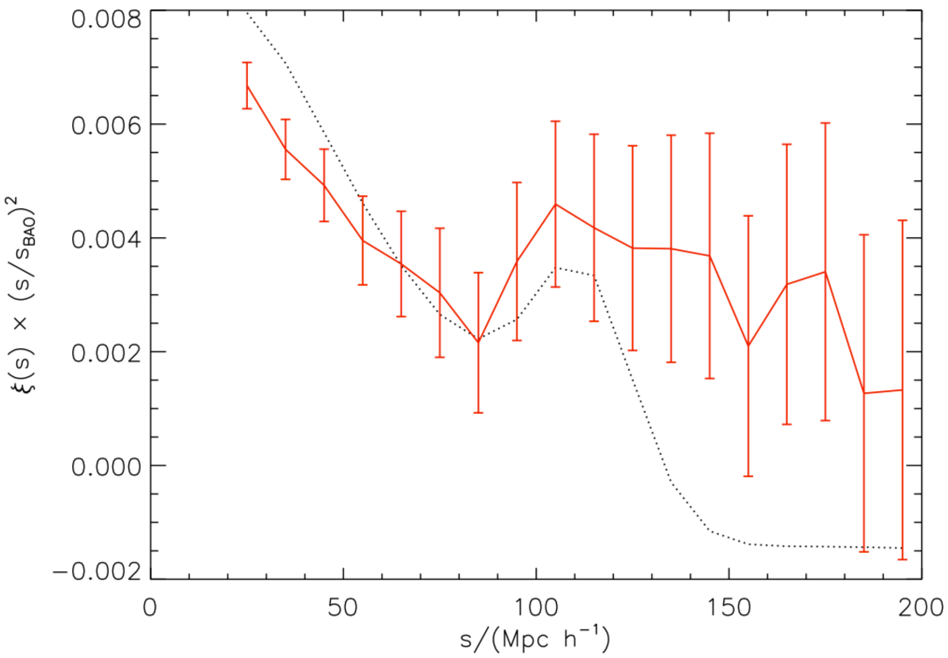
Effect of $\Omega_m h^2$





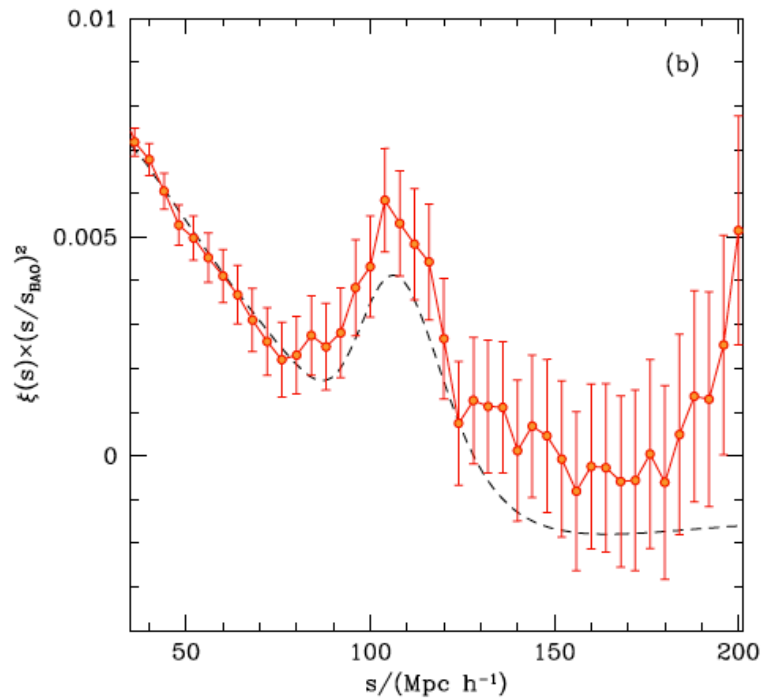
The whole covariance matrix is affected

SDSS Correlation function estimation



SDSS DR7:

my working sample



SDSS DR9

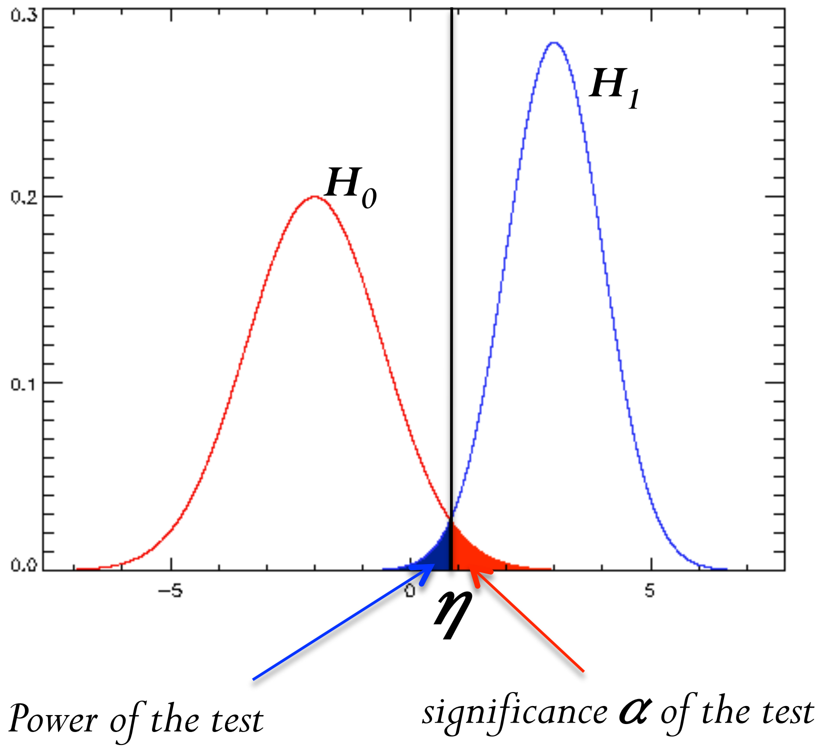
Sanchez et al (2012)

III) BAO detection

What does it mean to verify the existence of BAOs ?

Statistical Hypothesis testing

- Test between 2 hypotheses H_0 and H_1 from a measurement ξ using a test statistics $t(\xi)$



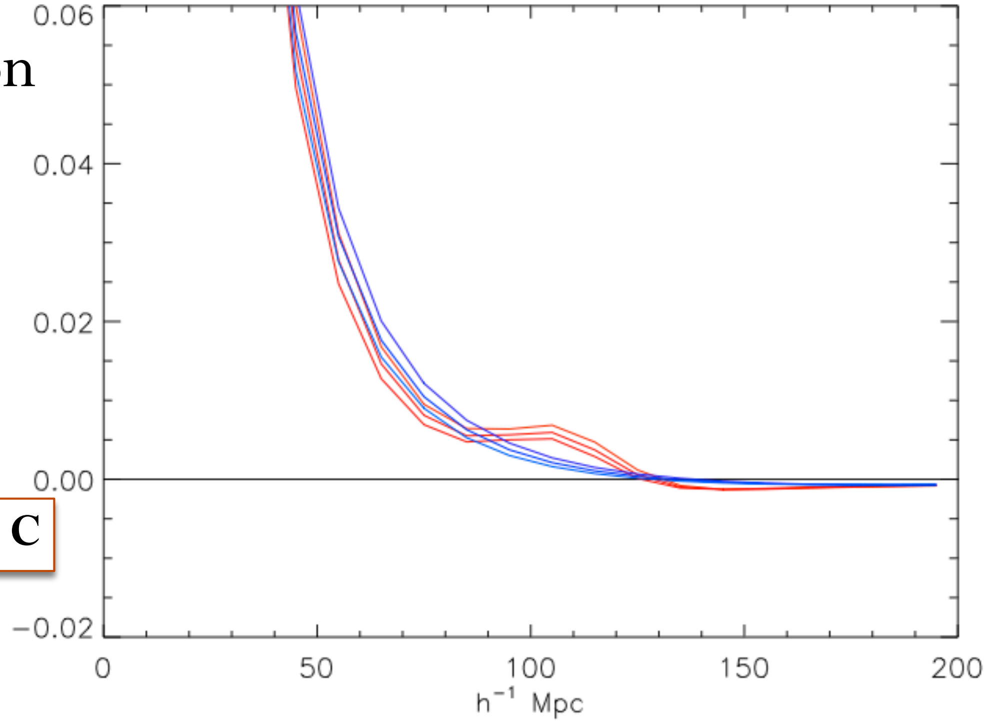
- Statistical test of size α
  - *If* $t(\xi) > \eta$ *then accept* H_1
  - *If* $t(\xi) \leq \eta$ *then accept* H_0

- More common in cosmology:

p-value α under H_0 that $t(\xi)$ > observed value

BAO detection with classical χ^2 method (I)

- If measurement is the correlation function:



$$\mathcal{H}_0 : \exists \theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{noBAO,\theta}, C\right)$$

$$\mathcal{H}_1 : \exists \theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{BAO,\theta}, C\right)$$

ASSUMES CONSTANT C

- Define the χ^2 statistics (equivalent to likelihood)

$$\chi^2_{BAO,\theta} = \left\langle \hat{\xi} - \xi_{BAO,\theta}, C^{-1}(\hat{\xi} - \xi_{BAO,\theta}) \right\rangle \qquad \mathcal{L}_{BAO,\theta}(\hat{\xi}) \propto \exp\left(-\frac{1}{2}\chi^2_{BAO,\theta}\right)$$

$$\chi^2_{noBAO,\theta} = \left\langle \hat{\xi} - \xi_{noBAO,\theta}, C^{-1}(\hat{\xi} - \xi_{noBAO,\theta}) \right\rangle \qquad \mathcal{L}_{noBAO,\theta}(\hat{\xi}) \propto \exp\left(-\frac{1}{2}\chi^2_{noBAO,\theta}\right)$$

BAO detection with classical χ^2 method (II)

- Test statistic $\Delta\chi^2$ is a generalized likelihood ratio (NP lemma)

$$\begin{aligned}\Delta\chi^2 &= \min_\theta \chi^2_{noBAO,\theta} - \min_\theta \chi^2_{BAO,\theta} \\ &= -2\ln\left[\frac{\max_\theta \mathcal{L}_{noBAO,\theta}}{\max_\theta \mathcal{L}_{BAO,\theta}}\right]\end{aligned}$$

- Significance can be estimated as $\sqrt{\Delta\chi^2}.\sigma$
- However:
  - $\sqrt{\Delta\chi^2}.\sigma$ is only an approximation
  - Gives very wrong results for model-dependent $\boldsymbol{C_\theta}$

BAO detection with Δ*l* method

- Brute force method to get correct *p*-value (=significance)

$$\mathcal{H}_0 : \exists \theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{noBAO,\theta}, C_{noBAO,\theta}\right)$$

Model-dependent $\mathbf{C}_\theta$

$$P(\Delta\chi^2 \geq x \,|\, \mathcal{H}_0, \theta)$$

$$p(x) = \max_{\theta \in \Theta} P(\Delta\chi^2 \geq x \,|\, \mathcal{H}_0, \theta)$$

- Replace test statistic $\Delta\chi^2$ by Δl

→ $\mathbf{\Delta l}$ **=** generalized likelihood ratio for model-dependent $\mathbf{C}_\theta$

$$\begin{aligned}\Delta l &= \min_\theta l_{noBAO,\theta} - \min_\theta l_{BAO,\theta} \\ l_{BAO,\theta} &= \chi^2_{BAO,\theta} + \ln |C_{BAO,\theta}| \\ l_{noBAO,\theta} &= \chi^2_{noBAO,\theta} + \ln |C_{noBAO,\theta}|\end{aligned}$$

Results for BAO detection under H_1 simulations

- Comparison of 3 methods for 2 different cases of covariance matrices

$$\mathcal{H}_0 : \exists \theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{noBAO,\theta}, C\right)$$
$$\mathcal{H}_1 : \exists \theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{BAO,\theta}, C\right)$$

$$\mathcal{H}_0 : \exists \theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{noBAO,\theta}, C_{noBAO,\theta}\right)$$
$$\mathcal{H}_1 : \exists \theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{BAO,\theta}, C_{BAO,\theta}\right)$$

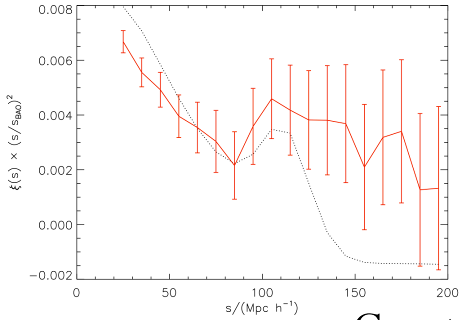
	Classical χ^2 method (wrong)	**$\Delta\chi^2$ statistic with correct significance**	**Δl method**
Constant covariance matrix	*2.21σ*	*2.0σ*	*2.0σ*
Model-dependent covariance matrix	*2.32σ*	*1.59σ*	*1.96σ*

IV) BAOs for cosmological parameter constraints

What is the effect of model-dependent C_θ for parameter constraints

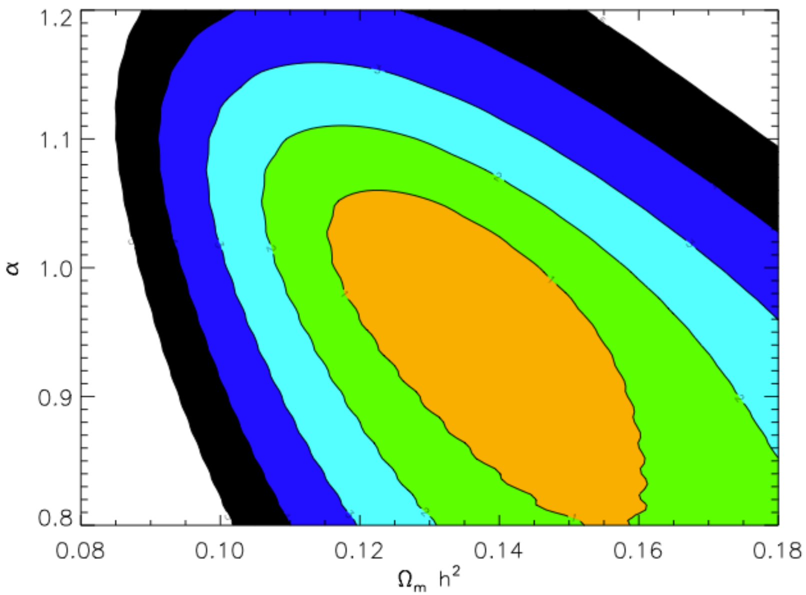
$$\mathcal{H}_1 : \exists \theta \in \Theta \text{ s.t. } \hat{\xi} \sim \mathcal{N}\left(\xi_{BAO,\theta}, C_{BAO,\theta}\right)$$

Parameter constraints with SDSS DR7 LRG

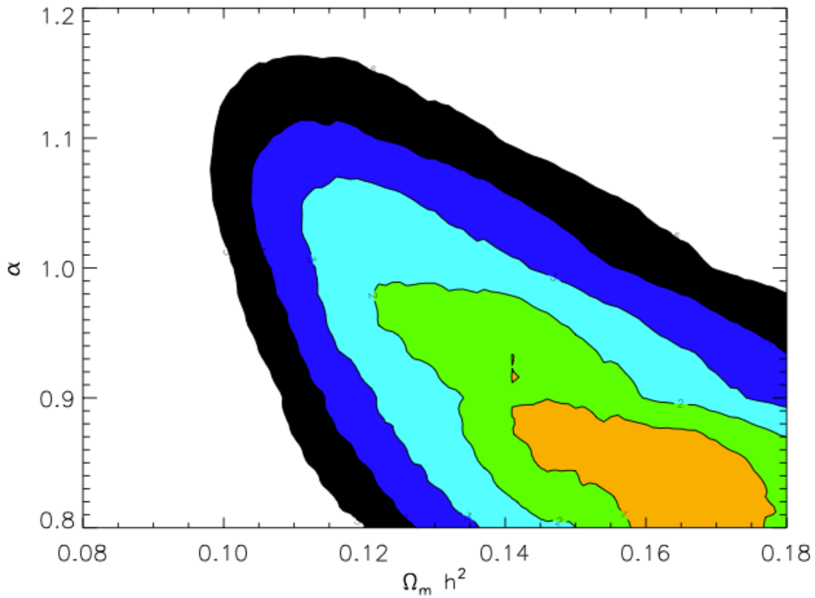


$$\mathcal{L}_\theta \propto |C_\theta|^{-1/2} \exp\left\{-\frac{1}{2}\left\langle \hat{\xi} - \xi_\theta, C_\theta^{-1}(\hat{\xi} - \xi_\theta)\right\rangle\right\}$$

Constant covariance



Model-dependent covariance



- Constraints are **tighter** with model-dependent $\boldsymbol{C_\theta}$
- **Non negligible shift** in the maximum likelihood

Conclusions

Conclusions

- New procedure for obtaining realistic model-dependent C_θ
- New method for BAO detection (Δl method)
  - Rigorous
  - Works even with model-dependent C_θ
- Consequence of model-dependent covariance C_θ for SDSS DR7 LRG sample:
  - **Can have a non-negligible effect, should be tested for future surveys like BOSS**

Thank you !

New procedure for simulations

- **New procedure to compute model-dependent C_θ**

  - Generate 2000 simulations for each value $\Omega_m h^2=0.08, 0.105, 0.13, 0.155, 0.18$

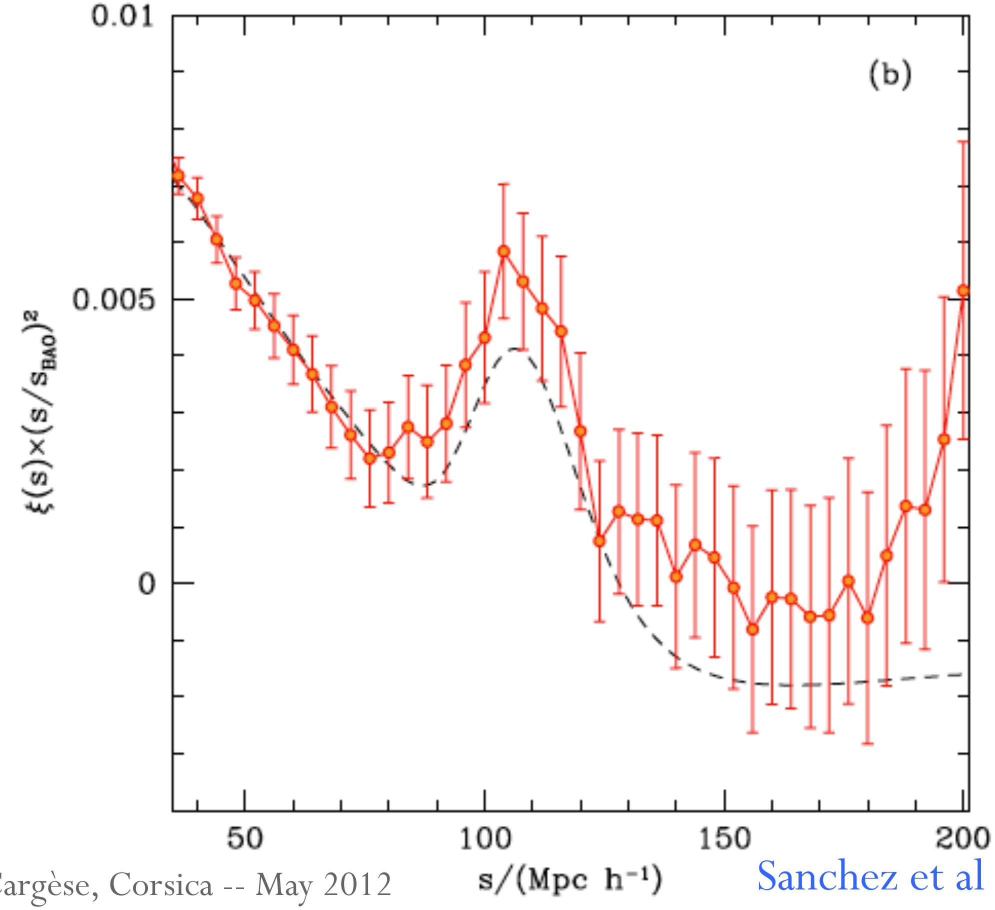
  - Geometric parameter α taken into account by introducing a selection function ϕ_α $(0.8< \alpha <1.2)$

  - b is well approximated by a factor b^4 in the covariance matrix

→ when changing b_1=2.5 to b_2=3.0 we find that $\boldsymbol{(b_2/b_1)^4\, C_1}$ is 10 times closer to $\boldsymbol{C_2}$ than $\boldsymbol{C_1}$

Correlation in BOSS DR9

- Public release expected in July 2012
- 6σ BAO detection !



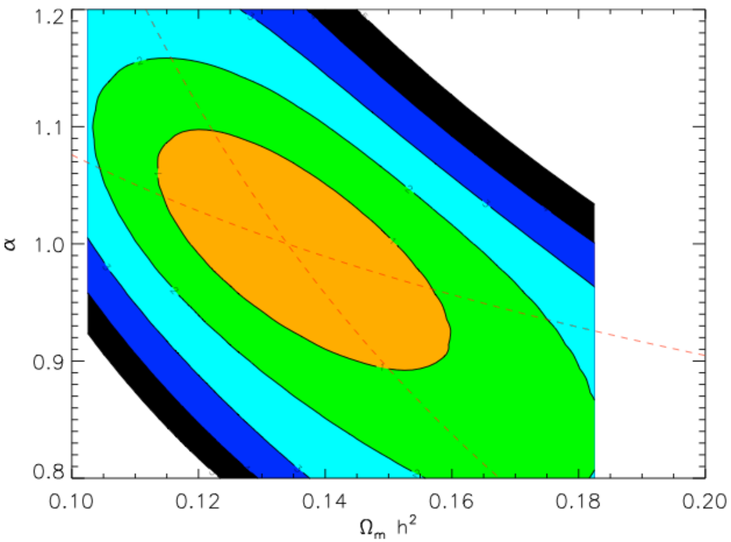
Sanchez et al (2012)

Effect of model-dependent covariance matrix in constraints

- Cosmological parameter constraints

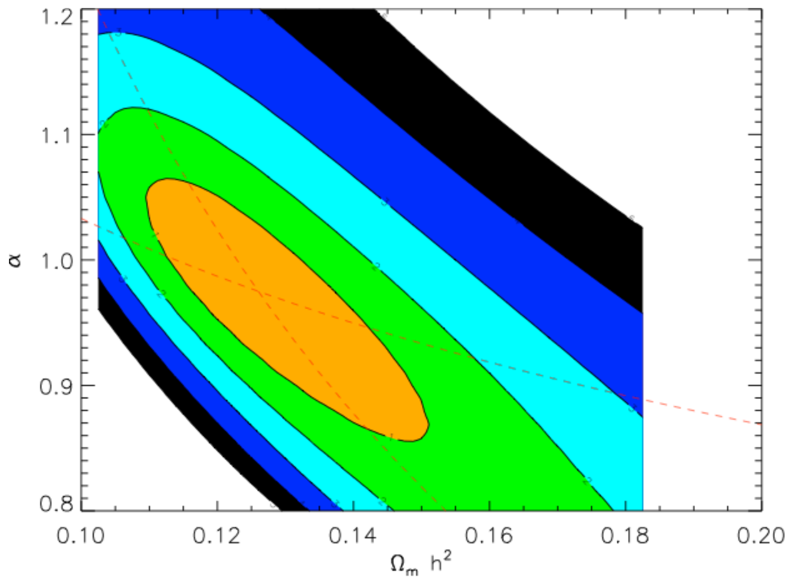
$$p(\Omega_m h^2, \alpha \,|\, \hat{\xi}) \propto \int \mathcal{L}_{BAO,(\Omega_m h^2, \alpha, B)} dB$$

- Example with expected correlation from simulations



Constant covariance

$$\begin{aligned} \alpha &= 0.995 \pm 0.070 \\ \Omega_m h^2 &= 0.134 \pm 0.015 \end{aligned}$$



Model-dependent covariance

$$\begin{aligned} \alpha &= 0.976 \pm 0.070 \\ \Omega_m h^2 &= 0.126 \pm 0.014 \end{aligned}$$

BAO detection with classical χ^2 method (II)

- The test statistic $\Delta\chi^2$ is a generalized likelihood ratio

$$\begin{aligned}\Delta\chi^2 &= \min_\theta \chi^2_{noBAO,\theta} - \min_\theta \chi^2_{BAO,\theta} \\ &= -2\left[\max_\theta \ln\left(\mathcal{L}_{noBAO,\theta}\right) - \max_\theta \ln\left(\mathcal{L}_{BAO,\theta}\right)\right] \\ &= -2\ln\left[\frac{\max_\theta \mathcal{L}_{noBAO,\theta}}{\max_\theta \mathcal{L}_{BAO,\theta}}\right]\end{aligned}$$

- Large values of $\Delta\chi^2$ favor $\boldsymbol{H_1}$

→ significance is proba under $\boldsymbol{H_0}$ that $\Delta\chi^2 >$ observed value

- Given some assumptions:

$$\Delta\chi^2 \leq X^2 \text{ with } X \sim \mathcal{N}(0,1) \text{ under } \mathcal{H}_0$$

- Significance can be estimated as $P(X^2 > \Delta\chi^2)$ i.e. $\sqrt{\Delta\chi^2}.\sigma$

Our new method for BAO detection (II)

- Change of the statistic
  - $\Delta\chi^2$ is not a generalized likelihood ratio for model-dependent covariance matrix

$$\mathcal{L}_{BAO,\theta} \propto |C_{BAO,\theta}|^{-1/2} \, e^{-\frac{\chi^2_{BAO,\theta}}{2}}$$

$$\mathcal{L}_{noBAO,\theta} \propto |C_{noBAO,\theta}|^{-1/2} \, e^{-\frac{\chi^2_{noBAO,\theta}}{2}}$$

  - We use Δl instead of $\Delta\chi^2$ to keep a generalized likelihood ratio

$$\begin{aligned} \Delta l &= \min_\theta l_{noBAO,\theta} - \min_\theta l_{BAO,\theta} \\ l_{BAO,\theta} &= \chi^2_{BAO,\theta} + \ln|C_{BAO,\theta}| \\ l_{noBAO,\theta} &= \chi^2_{noBAO,\theta} + \ln|C_{noBAO,\theta}| \end{aligned}$$